An Effective Tool for the Management of Stock Portfolio Using Variance-Covariance Approach of Value at Risk Models

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Abstract
Risk is a measure of how volatile an asset's returns are. Exposure to this volatility can lead to a loss in one's investments. Investors and their advisors spend a significant portion of their time researching the best investment options to maximize return and minimize risk. For this reason tools are used not only to passively measure and report risk, but also to defensively control or actively manage it. This paper discusses the implementation of an easy to use financial portfolio management tool based on the variance-covariance approach of Value-at-Risk models, with the aim to minimize risk in a stock portfolio. It estimates the maximum potential loss for a given probability and time horizon. The tool is built as a web application that can extract live trading values from the Nigeria Stock Exchange, calculates the value an investor can lose given a confidence level.

Keywords: *Risk, Stock Portfolio, Variance-Covariance, Value at Risk.*

1: Introduction

Portfolios, which are a grouping of financial assets such as stocks, bonds and cash equivalents, as well as their funds counterparts, including mutual, exchange-traded and closed funds, may be held by individual investors and/or managed by financial professionals, banks and other financial institutions. In a portfolio, one could have a collection of assets that belongs to different, or a specific asset class, such as stocks, real estate. Asset allocation is a problem faced by every investor. When making investment decisions, an investor has to seek a balance between risk and returns (Cho, 2008). One of the most important tasks of financial institutions is evaluation of exposure to market risks, which arise from variations in prices of equities, commodities, exchange rates, and interest rates.

According to (Spaseski, 2017), for investors, risk is about the odds of losing money, and VaR is based on that common sense fact. He further posited that by assuming that investors care about the odds of big losses, VaR can be used to answer the questions, "What is my worst-case scenario?" or "How much could I lose in a really bad month?"

As reported in (Holton, 2003) and (Jorion, 2000), there have been several large publicised losses in the 1990s, all of which have highlighted the need for accurate risk measure and control. The adequate representation and management of these risks is, therefore, a critical task for the success of a business. The dependence on market risks can be measured by changes in the portfolio value, or profits and losses. A commonly used methodology for estimation of market risks in a portfolio is the Value at Risk (VaR) (Khindanova, 1998).

A typical approach in risk management, as stated by (Uryasev, 2000), is to estimate and control the VaR with a specified confidence level, such as 0.95, 0.99, or 0.999. VaR is estimated for various periods, depending upon the risk management objectives – short term VaR is estimated usually for one day or two weeks, longer may include one, two or five years.

VaR is considered a popular measure of risk as it provides users with a summary measure of market risk using the same units as the portfolio's bottom line; e.g. GBP, USD or EUR depending upon the base currency of the portfolio. What this means is that VaR can also be communicated to a relatively non-technical audience, such as managers or shareholders (Kaura, 2011). According to (Holton, 2014) a value-at-risk measure must be implemented for it to be useful.

In this paper we describe and discuss the implementation of an easy-to-use tool using the variance-covariance approach of VaR models in the management of investment portfolios according to the investor's risk modelled by supposing that an investor choosing between several portfolios with identical expected returns will prefer that portfolio which minimizes risk; represented by a covariance estimator of the daily returns on assets. We demonstrate how the system can be used to calculate the VaR of any given stock portfolio.

2: Literature Review

2.1 Value at Risk

As posited by (Kaura, 2011), before looking at "Value at Risk" (VaR), we need to firstly define what risk is and, secondly, why we require a method to measure it. Risk is a measure of how volatile an asset's returns are. Exposure to this volatility can lead to a loss in ones investments. For this reason tools are used not only to passively measure and report risk, but also to defensively control or actively manage it. VaR is a popular method which regulators use to assess risk. When managed properly, VaR can provide a controlled way of getting high returns on ones investments. Modern portfolio theory models the return of an asset as a random variable and a portfolio as a weighted combination of these assets. This implies that the return of a portfolio is thus also a random variable and consequently has an expected value and a variance. Risk in this model is normally identified with the variance of portfolio return.

VaR is a measure of the worst expected loss that a firm may suffer over a period of time that has been specified by the user, under normal market conditions and a specified level of confidence. It is the expected loss of a portfolio over a specified time period for a set level of probability[1]

[1] http://www.yieldcurve.com/mktresearch/learningcurve/learningcurve3.pdf

VaR is defined as the pth percentile of portfolio return at the end of the planning horizon and for low p values (e.g. 1, 5 or 10) it can be thought of as identifying the "worst case" outcome of portfolio performance (Puelz, 2001). To put this into perspective, one might say that the VaR of their portfolio is N100,000.00 at the 95% confidence level with the target horizon set to one week. This means that there is a 5 out of 100 chance that the portfolio will lose over N100,000.00 within the target horizon under normal market conditions. The VaR of the portfolio is the loss that will not be exceeded in 95% of the cases, i.e. the lower 5% of returns which is equal to N100,000.00. VaR measures the potential loss in market value of a portfolio using estimated volatility and correlation. The "correlation" referred to is the correlation that exists between the market prices of different instruments in a bank's portfolio. This measure may be obtained in a number of ways, using a statistical model or by computer simulation
Formally, given a confidence level $\alpha \in (0,1)$, value-at-risk (VaR) of the portfolio at the confidence level α is the infimum value of the distribution of a random variable, L, and is given by the smallest number l such that the probability that the loss L exceeds l is at most $(1-\alpha)$. According to (Engemann, Gil-Lafuente & Merigó-Lindahl, 2012) usually this probability is taken to be 0.05 or less.

Mathematically, if L is the loss of a portfolio, then $\mathrm{VaR}_\alpha(L)$ is the confidence level α-quantile, i.e.

$$\mathrm{VaR}_\alpha(L) = \inf\{l \in \mathbb{R} : P(L > l) \le 1-\alpha\} = \inf\{l \in \mathbb{R} : F_L(l) \ge \alpha\}. \quad (1)$$

where $F_L(l)$ denotes the distribution function of L.
From equation 1 above we can see that VaR has three components which include a time period (day, month, or year), a confidence level (95% or 99%) and a loss amount (or loss percentage). We can therefore use VAR to answer questions such as; how much can I loose, at a 95% or 99% confidence level over the next day, month, year and what is the maximum percentage I can lose, with 95% or 99% confidence level, over the next year?

2.2 Measurement of VaR
(Puelz, 2001) identified three basic techniques employed to measure VaR: mean-variance, historical simulation and Monte Carlo simulation.

The variance covariance method uses information on the volatility and correlation of stocks to compute the VaR of a portfolio. The volatility of an asset is a measure of the degree to which price fluctuations have occurred in the past and hence expected to occur in the future. Correlation is a measure of the degree to which the price of one asset is related to the price of another (Kaura, 2011).

The historical simulation VaR measurement technique uses historical returns under a simulation approach. it works by keeping a record of daily profit and loss of the portfolio. The VaR of the portfolio is the loss that will not be exceeded in α % of the cases, ie. The lower (1−α)% of returns, where α is the confidence level. Intra-day, daily, weekly, monthly or quarterly historical return data are used as scenarios or possible future realizations to directly calculate the portfolio's VaR.

The Monte Carlo Simulation technique for measuring VaR, allows for complete flexibility with regard to security return distributions. In addition, the scenario sample set is not limited by historical realizations as securities are priced based on algorithms and/or heuristics imbedded in the model (Puelz, 2001).

2.3 Variance-Covariance Approach of VAR Models

The variance-covariance approach (also called parametric approach) for VaR, which is adopted in this work, assumes that stock returns are normally distributed, the correlations between risk factors are constant and the delta (or price sensitivity to changes in a risk factor) of each portfolio constituent is constant. In other words, it requires that we estimate only two factors - an expected (or average) return and a standard deviation (SD).

Figure 1 shows normal distribution with value of x -2,326 SD. Probability for losses greater than x is shown as an area under the normal curve, left of x. This area is 1% of total area under the curve, so there is confidence of 99% that losses won't exceed -2,326 SD.

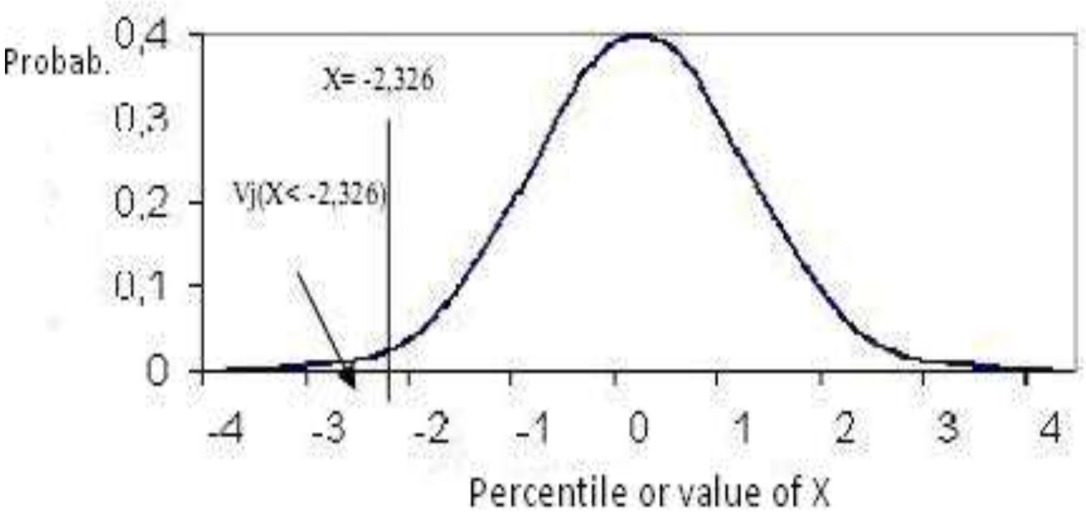


Figure 1: Percentile values and their probabilities with normal distribution (Ziković's in Čorkalo, 2011)

When measuring VaR only downward price changes are considered, or price changes that exceed some multiple of SD. Negative price change (in percentage) that corresponds to 1.65 SD gives confidence of 95% that loss won't exceed given value. And 2.33 SD gives confidence of 99% (Best, 1998). Finally VaR for portfolio can be calculated using the following formula:

$$VaRp = (Z\ V\ P) \quad (2)$$

where Z is standard value (calculated from confidence level), V is the volatility or standard deviation of asset/portfolio, and P is position (portfolio) value.

Generally VaR will not be calculated for a single position, but a portfolio of positions. In such a case we will require the portfolio volatility[2]. The portfolio volatility of a two-asset portfolio based on the volatility of each instrument in the portfolio (X and Y) and their correlation with one another is given by equation 3:

$$Vol_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2}} \quad (3)$$

[2] https://financetrain.com/

where: w_1 is the weighting of the first asset, w_2 is the weighting of the second asset, σ_1 is the standard deviation or volatility of the first asset, σ_2 is the standard deviation or volatility of the second asset, ρ is the correlation coefficient between the two assets.
The correlation coefficient between two assets, which measures how stocks vary together, uses the covariance between the assets in its calculation. The standard formula for covariance is shown at (4):

$$Cov(X,Y) = \frac{1}{n-1}\sum_{i=1}^{n}[(X_i - \overline{X}).(Y_i - \overline{Y})] \tag{4}$$

where the sum of the distance of each value X and Y from the mean is divided by the number of observations minus one. The covariance calculation enables us to calculate the correlation coefficient, shown as (5):

$$Cor(X,Y) = \frac{Cov(X,Y)}{\sigma_X.\sigma_Y} \tag{5}$$

where σ is the standard deviation of each asset. However, if there are more than two financial assets in the portfolio, then correlation and covariance matrices are needed to solve equations (Kulali, 2016).

$$\sigma_P^2 = [w_1 \dots w_n]\begin{bmatrix}\sigma_{11} & \dots & \sigma_{1n}\\ \vdots & & \vdots \\ \sigma_{n1} & \dots & \sigma_{nn}\end{bmatrix}\begin{bmatrix}w_1\\ \vdots \\ w_n\end{bmatrix} = w'\sum w \tag{6}$$

where w is the vector of the weights of the n assets, w' is the transpose vector of w and $\sum$ is the covariance matrix of the n assets.

This is the reason why this method is also known as Variance Covariance method. The generalised formula to calculate standard deviation of portfolio (position) is therefore given as:

$$\sigma_p = \sqrt{\sum_{i=1}^{n}(w_i^2.\sigma_i^2) + 2\left(\sum_{i=1}^{n}\sum_{j=1}^{n}(w_i.\sigma_i.w_j.\sigma_j p_{ij})\right)} \tag{7}$$

Where "σ_p" is a standard deviation of portfolio, "σ_i" is a standard deviation of stocks, "w_i" is a weight of stocks in a portfolio and "ρ_{ij}" is a correlation coefficient between stocks i and j.

The VaR will then be given by:

$$VaR_{1-\alpha} = P \times \sigma_P \times Z_\alpha \tag{8}$$

where VaR (1 – α) is the estimated VaR at the confidence level 100 × (1 – α)% and Z_α represents the number of standard deviations on the left side of the mean, at the required standard deviation, σ_p. P is the portfolio value[3].

[3] https://financetrain.com/

The daily price return which is the percentage in the price of an asset today relative to its price yesterday can be calculated using the logarithmic return, r_t, in equation 9.

$$r_t = \ln\left({}^{P_t}/_{P_{t-1}} \right) = \ln P_t - \ln P_{t-1} \quad (9)$$

where r_t is the log price return on day t, P_t is the price (closing price) of the security (asset) on day t and P_{t-1} is the price (closing price) of the asset on the trading day before and ln is the natural logarithm operator. Therefore, we can take the difference of the log prices to calculate log returns.

The multi-period returns can be calculated by taking the sum of the daily logarithmic returns (Ang, 2015). That is,

$$r_{1\ to\ T} = r_1 + r_2 + r_3 + \cdots + r_T$$
$$= \sum_{t=1}^{T} r_t \quad (10)$$

For example, to calculate the VaR for a single asset, we would calculate the standard deviation of its returns, using either its historical volatility or implied volatility. If a 95% confidence level is required, meaning we wish to have 5% of the observations in the left-hand tail of the normal distribution, this means that the observations in that area are 1.645 standard deviations away from the mean. Consider the following statistical data for a stock, calculated using one year's historical observations.

(Nominal: N10 million; Price: N100; Average return: 7.35%; Standard deviation: 1.99%).
The VaR at the 95% confidence level is 1.645 x 0.0199 = 0.032736. The portfolio has a market value of N10 million, so the VaR of the portfolio is 0.032736 x 10,000,000 = N327,360. So this figure is the maximum loss the portfolio may sustain over one year for 95% of the time.

For a two-asset portfolio let us assume that we want to calculate Parametric VaR at a 95% confidence level over a one-day horizon on a portfolio composed of two assets with the following assumptions: P = N100 million, w_1 = 50%, w_2 = 50%, σ_1 = 3%, σ_2 = 5%, ρ = 30%

$$VaR_{95\%} = 100000000\ x \sqrt{0.5^2 x 0.3^2 + 0.5^2 x 0.05^2 + 2x0.5x\ 0.5\ x\ 0.03x0.05x0.30.}\ x\ 1.645$$

$VaR_{95\%}$ = N5,393,493.00

3: Methodology

3.1 Requirements of the System

1. Input Requirements: for the estimation to be carried out successfully, the system will need to capture the following:

Initial weightings of assets in the portfolio- number of shares held of each asset.
Required return - a non-negative percentage figure
Confidence level –a non-negative percentage figure
Transaction costs –a non-negative percentage value for each asset.

2. Compute the VAR of a Portfolio - the system's main goal is to use the inputs given by the user to estimate the maximum potential loss of an investor for a given confidence level and time horizon.

3. **Outputs**: Display result of the computation; report of value-at-risk for the portfolio.

3.2 The Database

The database forms the core of the application; the database will store the following data as indicated in the database schema/relation:

```
Asset_database (
AssetID INT NOT NULL,
Name VARCHAR (20)           /* Names of assets in the stock exchange.*/
Symbol VARCHAR (20)         /* symbols */
Histprice FLOAT,            /* Historical price data for each asset */
Cov FLOAT                   /* Covariance values of assets   */
Transcost FLOAT             /* Transaction cost values   */
PRIMARY KEY (AssetID)
```

3.3 VARestimator

The **VARestimator** will return the value of the VaR calculated. Below is the UML diagram for the stock portfolio management system.

VARestimator
-rateOfReturn: double -confidence: double -portfolioValue: double -assetWeights: double
+getrateOfReturn(): double +getConfidence():double +getportfolioValue():double +getAssetWeights(): double

getrateOfReturn(): This method calculates the returns on the prices using equation 8.

$r_{log} = In(p_{today} / p_{yersterday})$

where p_{today} =Today's Closing Price

$p_{yersterday}$ = Previous trading day's Closing Price

getConfidence(): this method sets the confidence level.

getportfolioValue(): this is used to get the total amount invested in the portfolio.

getAssetWeights(): this methods calculate the weight of each stock using:

$$Weight = \frac{Value\ of\ Stock}{Total\ Portfolio\ Value} x100$$

3.4 Algorithm VARestimator:

To estimate the maximum potential loss of a portfolio with n number of assets, the system follows the steps below:

Begin:

[1]: *Obtain the closing price of the day for each stock in the portfolio.*

[2]: *Calculate the returns on the prices using logarithmic return*

[3]: *Calculate the standard deviation of the historical observation of the returns.*

[4]: *Enter the amount invested in each stock.*

[5]: *Calculate the weight of each stock.*

[6]: *Calculate the correlation coefficient between the stocks*

[7]: *Calculate the standard deviation (SD) of the portfolio*

***[8]:** Enter the confidence level as standard normal distribution quantile, since confidence level is mainly between 95% to 99%, we assign the following values which has already been computed by looking up the normal distribution table as follows:*

95% - 1.645
96% - 1.755
97% - 1.970
98% - 2.110
99% - 2.350

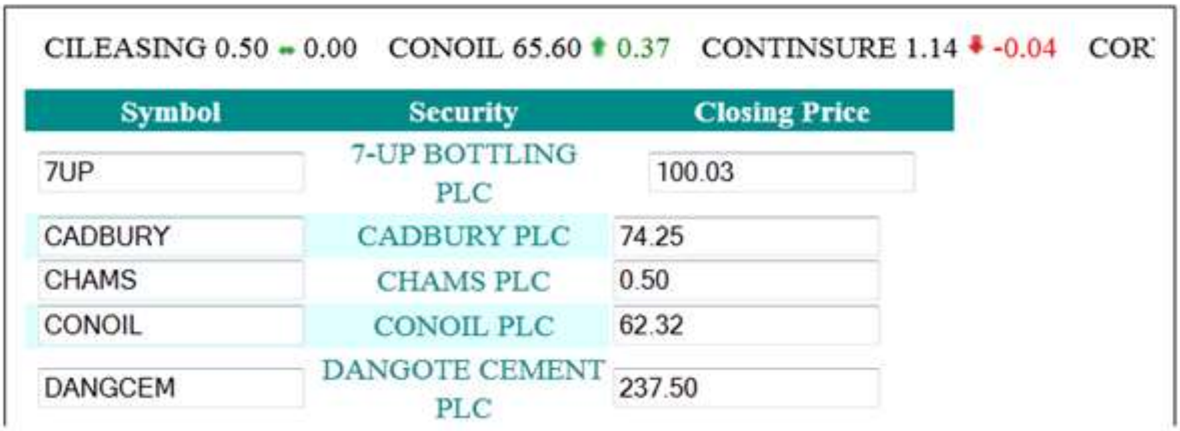
***[9]:** Calculate the VaR at the confidence level*

Stop.

4: Implementation and Discussion of Results

The system is simulated using hypothetical portfolio, with a number of assets. The system is connected to the Nigerian Stock Exchange API (NSEAPI)[4] to obtain real time data of historical prices. The end user interacts with the system directly through the graphical user interface. This component is used to capture the inputs from the user and provide the means to start off the computation procedure and display the results.

Once the system is launched, the home page displays the available stocks and their closing prices, real-time performance of each stock as shown in figure 2. The closing price is displayed in real-time by the NSEAPI[5].



Symbol	Security	Closing Price
7UP	7-UP BOTTLING PLC	100.03
CADBURY	CADBURY PLC	74.25
CHAMS	CHAMS PLC	0.50
CONOIL	CONOIL PLC	62.32
DANGCEM	DANGOTE CEMENT PLC	237.50

Figure 2: Home Page

[4] http://www.nse.com.ng/market-data/data-products-and-services/real-time-data
[5] http://nseapi.com/

The database can be updated with the current prices by clicking the update button as shown in figure 3.

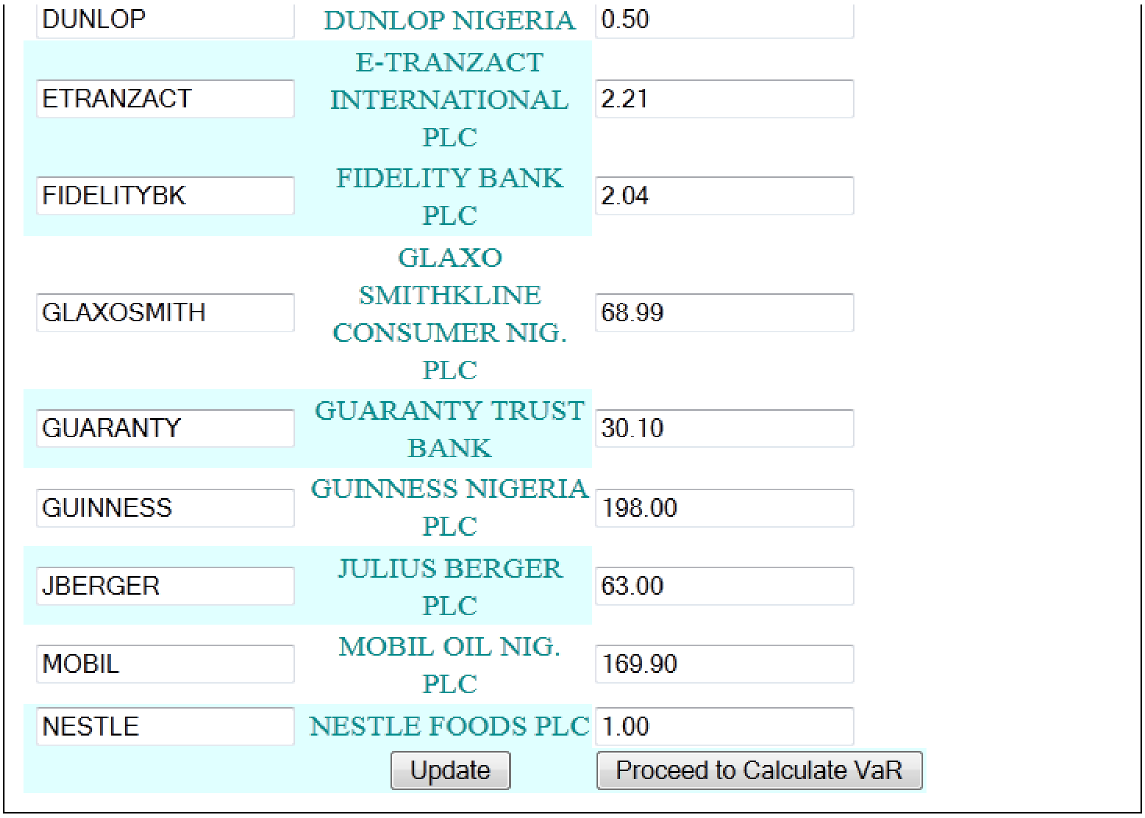


Figure 3: Updating Database

Figure 4 is the interface where the investor enters all the variables needed to calculate the value at risk.

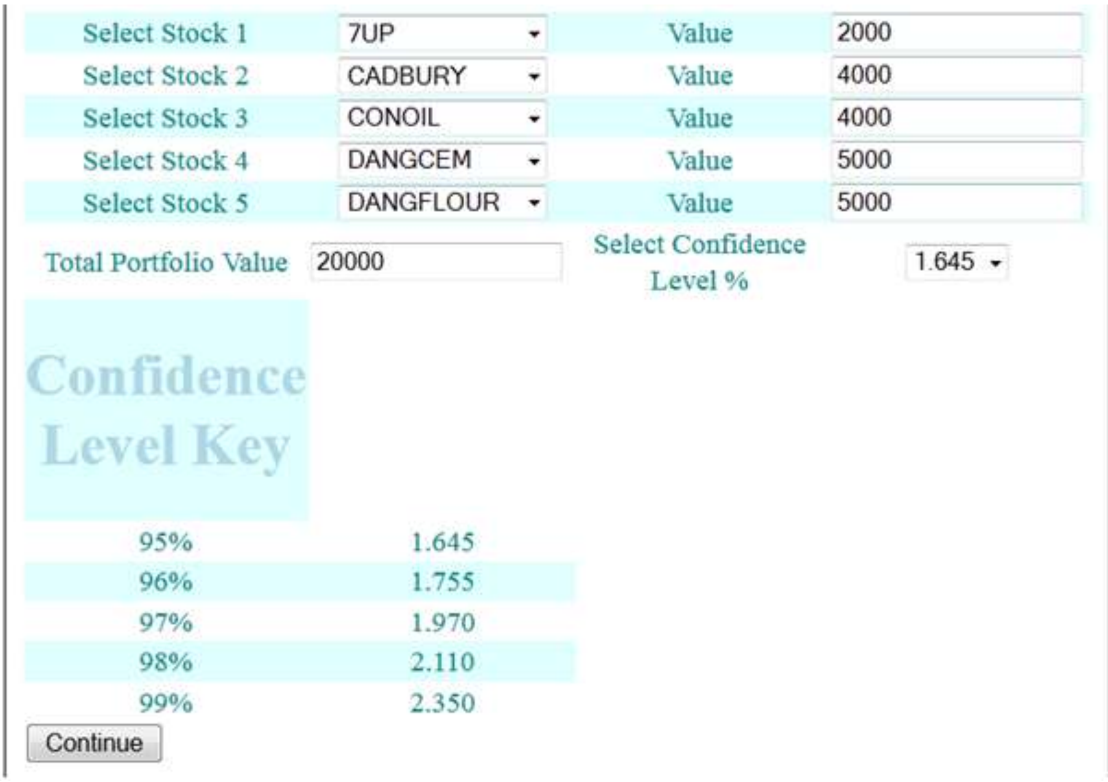


Figure 4: Calculate VaR Page

The result page (figure 5) shows the result of the Var calculations. Please note a negative Value at Risk indicates a 99% probability for a positive performance, which does not imply that the product is risk-free.

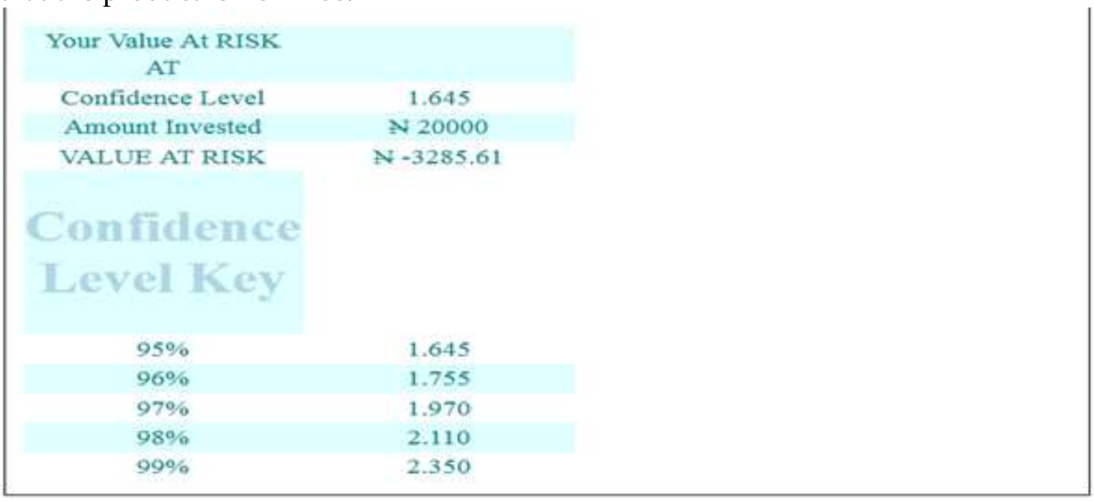


Figure 5: Result Page

5 Conclusion

In this paper we considered the implementation of an easy-to-use but highly effective tool which uses the variance-covariance approach of VaR models in the management of investment portfolios. The tool will be extremely useful to risk management according to the investor's risk modeled by helping an investor to choose between several portfolios with identical expected returns; in which case the investor might seek a portfolio that minimizes risk represented by a covariance estimator of the daily returns on assets. It has been built as a web application that extracts live trading values from the NSE, calculates the value an investor can lose given a confidence level. We demonstrated how the system can be used to calculate the VaR of any given stock portfolio.

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